AMUEM 2005 – International Workshop
on Advanced Methods for Uncertainty Estimation in Measurement
Niagara Falls, Ontario, Canada, 13 May 2005

Uncertainty Estimation in Noise Figure Measurements at Microwave Frequencies

N. Otegi, J.M. Collantes, M. Sayed[1]
Electricity and Electronics Department, University of the Basque Country, Apdo. 644, 48080 Bilbao, Spain
Phone: +34-94-601-2464, Fax: +34-94-601-3071, Email: jmcollan@we.lc.ehu.es
[1]Microwave & MillimiterWave Solutions, Santa Rosa, California, USA

Abstract *– In this paper, a specific simulation tool is developed in order to evaluate systematic error and measurement uncertainty associated to different methodologies for noise figure measurement of microwave circuits. The tool is based on Monte-Carlo algorithms and follows the recommendations of the IEC-ISO Guide to the Expression of Uncertainty in Measurement (GUM). Representative examples are given to illustrate the capabilities of the proposed tool to assist microwave engineers in the selection of the methodology and the measurement setup best suited for their needs.*

Keywords *– noise figure measurements, microwave circuits, uncertainty, systematic error.*

I. INTRODUCTION

Research and development in microwave instrumentation shows a growing tendency to develop instruments that integrate different types of measurements in a unique system, in an attempt to reduce and optimize the time and the required equipment needed for a complete device characterization. In particular, the integration of scattering parameter and noise figure measurement capabilities in a same setup (or even better in a same instrument box) has important benefits. On the one hand, basic device characterization (gain, match and noise figure) can be performed straight off, with a substantial gain in time that is especially worthwhile in production environments. On the other hand, thanks to the vector measurement and calibration, more accurate methodologies, including terms accounting for systematic errors, can be applied to improve the accuracy in the noise figure computation.

Errors affecting the noise-figure measurement have diverse sources and depend on the methodology that is used for noise figure calculation. The basic formulations of typical methodologies (*Y-factor*, *cold-source*) are often complemented with additional correction terms in order to eliminate errors arising from systematic effects, such as mismatch related effects, noise added by the receiver, etc. However, the correction terms used to remove systematic errors have their own uncertainty due to the incomplete knowledge of the required value of the correction. Besides, some systematic errors may not be taken into account by the expressions used for noise figure calculation. In addition, there is also error arising from random effects (such us connector variability, jitter…), uncertainty related with the limited accuracy of the measurement instruments; with the imperfect knowledge of the hot and cold noise temperatures, etc. The suitability of a particular methodology, having a specific level of corrections, is a function of the DUT and setup characteristics. In some cases, complicating the measurement process to include correction terms will not necessarily lead to more accurate results, while in other cases corrections may be crucial for achieving the desired accuracy.

Therefore, at the moment of designing these new and more performing equipments for noise figure characterization, some questions arise concerning the basic strategy and formulation to be used, the hardware characteristics of the setup, the real benefits of each systematic error correction, the acceptable compromise between accuracy and measurement complexity, etc.

Traditional uncertainty analyses associated to noise figure calculation were oriented to the scalar measurements of very particular methodologies (scalar *Y-factor*) [1] and are of little help in developing new instruments with more performing methodologies. In this paper, a rigorous uncertainty estimation tool has been developed with the objective of providing helpful information to instrument developers in order to address efficiently the above matters.

The tool is based on a Monte-Carlo simulation method that can be applied to any kind of noise figure characterization methodology. The uncertainty estimation procedures have been designed following the recommendations of the IEC-ISO Guide to the Expression of Uncertainty in Measurement (GUM) [2]. In particular, GUM recommendation on how to deal with systematic and random effects has been especially helpful in order to correctly identify the influence of systematic errors and measurement uncertainty. Differently from previous approaches [3], an independent formulation of the overall systematic error and the uncertainty will allow a better and more consistent comparison between the different characterization strategies.

Some basic concepts about uncertainty estimation according to GUM are summarized in the following section. Details on the modeling of the noise figure measurement setup and the calculation methodologies are given in section 3. Section 4 exposes the fundamental characteristics of the uncertainty estimation tool. Finally, application examples are given in section 5 in order to illustrate the capabilities of the proposed tool.

II. UNCERTAINTY ESTIMATION BASICS

GUM provides general rules for evaluating and expressing uncertainty associated with the conceptual design and theoretical analysis of methods and experiments in complex measurement systems, as the one under analysis. Thus, the noise figure uncertainty estimation tool has been developed according to basic concepts and notions from GUM. Some of the most important fundamentals utilized in this work are summarized in this section.

Measurement imperfections of diverse origin give rise to errors in the measurement result. Thus, in general, the result of a measurement is only an approximation or estimate of the value of the measurand. If the error is originated from a systematic effect it is called systematic error and it can be eliminated by the proper inclusion of a correction factor (at least in theory). According to [2], uncertainty is formally defined as a parameter associated with the result of a measurement that characterizes the dispersion of the values that could be reasonably attributed to the measurand. Error and uncertainty represent completely different concepts. Indeed, the uncertainty of a measurement result reflects the lack of exact knowledge of the value of the measurand. The result of a measurement after correction for systematic effects is still only an estimate of the value of the measurand because of the uncertainty arising from random effects and from imperfect knowledge of the correction factors for systematic effects.

The uncertainty is called standard uncertainty when it is expressed as a standard deviation (combined standard uncertainty if the result of a measurement is obtained from the values of other quantities). If all the quantities on which the result of the measurement depends are varied, the combined standard uncertainty can be evaluated by statistical methods. Let us consider a measurand Y that is determined from other quantities $X_1, X_2, ..., X_n$ through a function f, where f includes all corrections and correction factors that are taken into account.

$$Y = f(X_1, X_2, \ldots, X_n) \tag{1}$$

An estimate of the measurand y can be obtained from the arithmetical mean of a set of m independent observations of Y:

$$y = \overline{Y} = \frac{1}{m}\sum_{k=1}^{m} Y_k = \frac{1}{m}\sum_{k=1}^{m} f\left(X_{1,k}, X_{2,k}, \ldots, X_{n,k}\right) \tag{2}$$

This way of averaging, rather than using the arithmetical means of the individual observations $y = f\left(\overline{X_1}, \overline{X_2}, \ldots \overline{X_n}\right)$, may be preferable when f is a nonlinear function and correlation between variables need to be considered, as in the case of noise figure calculation.

Corrections for systematic effects may not always be convenient or even possible, either because of an unacceptable increase in time, calibration complexity, required equipment, etc. In some cases, attempts are made to take the systematic effect into account by "enlarging" the uncertainty assigned to the result. As recommended by [2], this practice has been avoided in the noise figure uncertainty estimation tool that is developed in this work. Avoiding the combination of systematic errors with measurement uncertainty provides more consistent information about the correct strategy for noise figure calculation. This will be clearly evidenced by the application examples.

III. NOISE FIGURE BASICS AND MEASUREMENT SETUP MODELING

A. Noise figure definition

The noise figure is a figure of merit that quantifies the degradation of the signal-to-noise ratio (SNR) between the input and the output terminals of a 2-port network due to the noise added by the network itself. Noise figure can also be seen as a measure of the amount of noise added by a 2-port network to the RF signal that passes through it.

The noise figure is defined as the ratio of the (SNR) at the input of a 2-port device to the SNR at the output, when the input noise is the available thermal noise power of a resistive termination at a reference temperature T_0 =290 K.

$$F = \left.\frac{S_i / N_i}{S_o / N_o}\right|_{T=T_0} \tag{3}$$

where S_i and S_o are the signal levels available at the input and the output of the device and N_i and N_o are the available noise powers, respectively.

The input noise is given by:

$$N_i = kT_0 B \tag{4}$$

where k is the Boltzmann constant and B is the considered bandwidth.

The output noise N_o can be expressed as a function of the noise generated by the device N_{add}:

$$N_o = N_{add} + G_{av} N_i \tag{5}$$

Gav is the available gain of the device, defined as:

$$G_{av} = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}|S_{21}|^2 \frac{1}{1-|\Gamma_{out}|^2} \tag{6}$$

S_{ij} being device S parameters, Γs the source reflection coefficient connected to the device and Γout its output reflection coefficient:

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1-S_{11}\Gamma_s} \tag{7}$$

Thus, equation (3) can be rewritten as:

$$F = \frac{N_{add} + kT_0 B G_{av}}{kT_0 B G_{av}} \tag{8}$$

which is the standard IEEE definition of the noise figure [4].

A significant property of the noise figure is that it depends on the source impedance connected at the input of the device, requiring a set of four independent parameters for describing such dependence. A commonly used expression for this characterization is:

$$F = F_{\min} + 4\frac{R_n}{Z_0}\frac{|\Gamma_s - \Gamma_{opt}|^2}{|1+\Gamma_{opt}|^2 (1-|\Gamma_s|^2)} \quad (9)$$

where Z_0 is the characteristic impedance and *Fmin*, *Rn*, real(*Γopt*) and imag(*Γopt*) are the four noise parameters.

B. Noise figure measurement

In any real noise figure characterization setup, the noise generated by the measurement system must be taken into account. If we consider the cascaded system composed by the Device Under Test (DUT) and the noise receiver, the DUT noise figure F_{DUT} can be de-embedded from the system noise figure *Fsys* by making use of the Friis formula [5]:

$$F_{DUT}(\Gamma_s) = F_{sys}(\Gamma_s) - \frac{F_{rec}(\Gamma_{out}) - 1}{G_{av}} \quad (10)$$

where *Frec* is the receiver noise figure when the reflection coefficient *Γout* is connected at its input.

Equation (10) is often referred to as the second-stage correction. Note that, if the DUT has an available gain large enough to make the second term of (10) negligible, then F_{DUT} becomes equal to *Fsys*. Otherwise, knowledge of all three terms is required to accurately determine the noise figure of the DUT.

The two basic formulations for the calculation of noise figure are considered here: the *Y-factor* technique and the *cold-source* technique. The bases of these two techniques are widely reported in the literature [6]-[7]. Some fundamentals on these procedures are summarized here.

In the *Y-factor* method, the noise figure is computed from two noise powers (*Nh, Nc*) measured with the noise source at its hot and cold temperatures (*Th, Tc*) respectively.

$$F = \left[\left(\frac{T_h}{T_0} - 1\right) - \frac{N_h}{N_c}\left(\frac{T_c}{T_0} - 1\right)\right] \bigg/ \left(\frac{N_h}{N_c} - 1\right) \quad (11)$$

Assuming that the cold temperature *Tc* is equal to the reference temperature T_0, the noise figure can be obtained from (12), which constitutes the simplest formulation for noise figure calculation.

$$F = \left(\frac{T_h}{T_0} - 1\right) \bigg/ \left(\frac{N_h}{N_c} - 1\right) \quad (12)$$

However, this calculated noise figure is the global noise figure of the system composed by the DUT and the noise receiver (*Fsys*) and not the noise figure of the DUT (F_{DUT}). In order to obtain F_{DUT}, second-stage correction must be applied, which requires the knowledge of *Frec*. For that, a calibration step is performed, where the noise source is directly connected to the receiver and *Frec* is calculated as described in expressions (11) or (12), from the measured hot and cold noise powers (*Nrec_h*, *Nrec_c*). Nevertheless, it must be pointed out that the receiver noise figure obtained in the calibration stage is *Frec(Γs)* and not *Frec(Γout)*, as demanded by the Friis formula. For *Frec(Γout)* to be computed, receiver noise parameters knowledge i.e., a noise calibration of the receiver, is required.

In addition to this approximation, available gain is usually substituted by insertion gain *Gins*, which requires only scalar measurements. Both gains are equal for perfect match conditions but they can diverge significantly when DUT output match degrades.

$$G_{ins} = \frac{N_h - N_c}{N_{rec\_h} - N_{rec\_c}} \quad (13)$$

With these two approximations, F_{DUT} is given by expression (14), which is the most extended noise figure measurement method, used by the most common commercially available noise figure meters.

$$F_{DUT} = F - \frac{F_{rec}(\Gamma_s) - 1}{G_{ins}} \quad (14)$$

The *cold-source* technique computes the noise figure from a single noise measurement (*Nc*) with a 50-Ohm source impedance, at room temperature, connected to the input of the DUT. For that, the gain-bandwidth product of the receiver (*BGrec*) and the available gain of the DUT have to be previously determined.

$$F = \frac{N_c}{KT_0 BG_{rec} G_{av}} - \left(\frac{T_c}{T_0} - 1\right) \quad (15)$$

Once again, assuming that the room temperature is equal to the reference temperature T_0, the simplest formulation for *cold-source* is:

$$F = \frac{N_c}{KT_0 BG_{rec} G_{av}} \quad (16)$$

Again, a calibration stage is required for removing the noise contribution of the receiver. Therefore, F_{DUT} can be approximated by expression (17).

$$F_{DUT} = \left(\frac{N_c}{KT_0 BG_{rec} G_{av}} - \frac{F_{rec}(\Gamma_s) - 1}{G_{av}}\right) \quad (17)$$

The *BGrec* term is also calculated in the calibration stage:

$$kBG_{rec} = \frac{N_{h\_rec} - N_{c\_rec}}{T_h - T_c} \quad (18)$$

The correct application of the second stage correction needs again the knowledge of the receiver noise parameters, in order to calculate *Frec(Γout)*.

C. Measurement setup modeling

The generic measurement setup (Fig. 1) has been mathematically modeled. The setup includes a noise source with each of its two temperature states (cold and hot *Tc, Th*) having a corresponding reflection coefficient *(Γs_c, Γs_h)*.

Two 2-port devices (input and output), defined by their *S* parameters, model any passive block connected at the input and output ports of the DUT. The noise receiver is characterized by its input reflection coefficient (*Γrec*), its gain-bandwidth product (*BGrec*) and its four noise parameters (*Fmin, Rn, Γopt*). Linearity and thermal drift of the receiver are also modeled through its 1 dB compression point (*P1dB*) and a thermal drift factor (*Dth*) respectively. Finally, the DUT is defined by its *S* parameters and by its four noise parameters (*Fmin', Rn', Γopt'*).

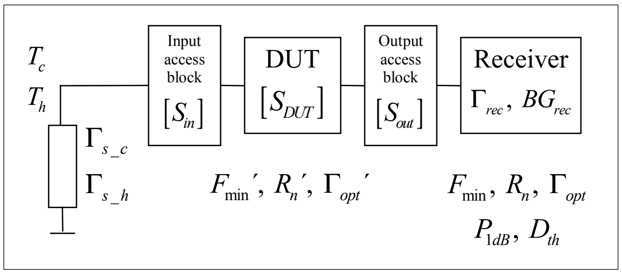

Fig. 1. Modeling of the measurement setup

The basic *Y-factor* and *cold-source* formulations represented by (12) and (16) can be complemented with the inclusion of correction terms to account for systematic effects, such as mismatch related effects, noise added by the receiver, etc. The most significant corrections that can be taken into account by the simulation tool are listed here:

- De-embedding the DUT noise figure from the whole system noise figure (second stage correction).
- Calculation of the DUT available gain required for a rigorous application of the previous de-embedding.
- Vector corrections accounting for any access network that may be placed at the input and output ports of the DUT (on-wafer probes, adapters, etc...).
- Differences between the cold temperature (*Tc*) and the reference temperature (T_0 *=290 K*).
- Variations in the reflection coefficient of the noise source from cold to hot temperatures (*Γs_c ≠ Γs_h*). Obviously, this effect cannot be completely removed from the *Y-factor* formulation, since that would require the knowledge of the DUT noise parameters, which are *a priori* unknown.
- Dependence of the receiver noise figure on the source termination through the set of four noise parameters (*Fmin, Rn, Γopt*).
- Corrections for thermal drift (*Dth*) and gain compression of the receiver (*P1dB*).

IV. UNCERTAINTY ESTIMATION TOOL

The measurement setup and the two methodologies with their different corrections have been mathematically represented in *Matlab*. A user interface has been developed to facilitate the comparative analysis among methodologies with different level of corrections for systematic effects, versus any parameter involved in the analysis.

The use of the term "true value" is carefully avoided by [2] because it represents an unknowable quantity. However, it is important to remember that the aim of this uncertainty estimation tool is to compare different strategies with different level of corrections to determine which one is more suitable for some given characteristics of DUT and measurement setup. For this reason, conventional "true" values are assigned to every parameter of the setup. As a consequence, the simulated noise power obtained from them is also considered as "true" value.

Standard uncertainties can be associated to any system parameter involved in the calculation. Then, noise figure is calculated a large amount of times (in the standard manner of Monte Carlo methods) from parameter values that are randomly chosen from a Gaussian distribution centered at the "true" value of each parameter. From these results a mean value *m* and a standard deviation σ are calculated. The standard deviation directly provides the combined standard uncertainty $u_c = \sigma$. The systematic error is estimated as: $e_s = m - F_{true}$, where F_{true} is the noise figure obtained from the "true" values. The tool can also calculate an expanded uncertainty multiplying the obtained standard deviation by a coverage factor *k* associated to a specific level of confidence, $U = k\sigma$.

V. APPLICATION EXAMPLES

As in [3], parameter data from a Q-band (33GHz - 50GHz) custom built noise meter is used in the examples. The DUT is a 44 GHz low noise amplifier for radio astronomy applications with the following characteristics: S_{21} = 10dB∠7.9º, S_{11}= 0.48∠102º, S_{22} = 0.41∠43º and $F \approx 3$ dB. Type B uncertainties are estimated for the proposed DUT and measurement setup. Typical standard uncertainties, given by manufacturer's specifications at Q band, are assumed for the *Excess Noise Ratio (ENR)*, for every variable associated to scattering measurements, for the receiver noise parameters and for the noise power measurements.

It is important to note that, the aim of the following examples is not to extract general results about the different methodologies for noise figure calculation, but to show the suitability of the developed tool for choosing the appropriate methodology as a function of DUT and setup characteristics. In particular, a comparative analysis among different methodologies with different levels of corrections is presented here. All the approaches are analyzed versus the DUT gain (S_{21} parameter), although any other element involved in the calculation could have been selected as a variable parameter.

The most basic methodology for noise figure measurement is considered first. This is a standard *Y-factor* technique in which only scalar power measurements are used (YF_1). The only systematic effect taken into account is the de-embedding of the second stage noise. YF_1 is a widely extended approach used by typical noise figure meters. The obtained systematic error and combined standard uncertainty are plotted versus DUT gain in Fig. 2. While the uncertainty remains constant with DUT gain, an expected increase of the systematic error for low gain can be observed. This systematic error is generally attributed to mismatch effects that cannot be corrected in YF_1 approach because of the lack of vector measurements. Advanced equipment that combines scattering parameters and noise figure measurements proposes a *Y-factor* methodology that includes a correction term based on the calculation of the available gain from the measured scattering parameters (YF_2) [8]. Results of the analysis for the YF_2 approach are plotted in Fig. 3. Contrarily as it would be expected, systematic error at low gain is significantly increased. A lighter increase of the uncertainty is also noticeable for low gain values. This result evidences that, in the noise figure measurement and calculation process, the combination of some systematic effects tends, in some cases, to compensate each other [9]. Therefore, including correction terms accounting for only one of them can lead to a higher systematic error. Actually, if an additional correction term is introduced in the *Y-factor* calculation, accounting for the dependence of the receiver noise on the source termination (YF_3), systematic error at low gain can be clearly improved (Fig. 4). This result confirms that these two systematic effects must be taken into account simultaneously in order to guarantee an improvement of the systematic error. It is important to observe in Fig. 4 the expected increase of the uncertainty at low gain, originated by the inclusion of the correction terms.

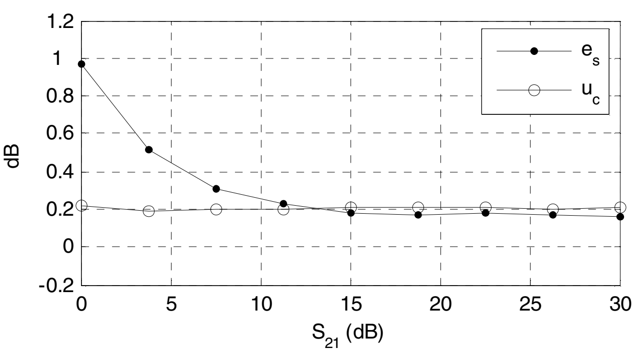

Fig. 2. *Y-factor* based methodology YF_1. Only correction for second stage noise is taken into account.

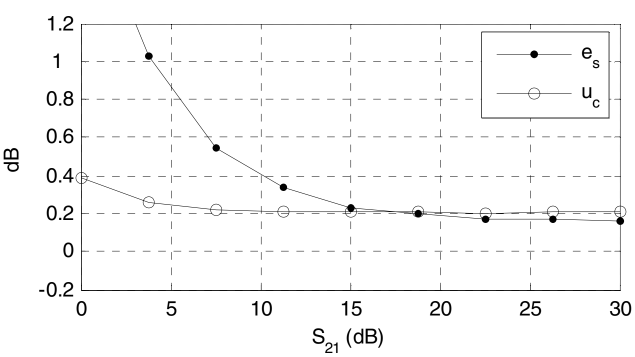

Fig. 3. *Y-factor* based methodology YF_2. Corrections account for second stage noise and DUT available gain.

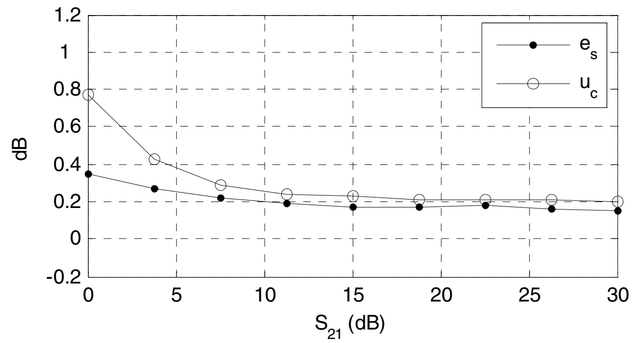

Fig. 4. *Y-factor* based methodology YF_3. Corrections account for second stage noise, DUT available gain and receiver noise figure variation with source termination.

Results from Fig. 2 to Fig. 4 show a constant and non negligible systematic error for high DUT gain in the three analyzed approaches (YF_1, YF_2 and YF_3). A term corresponding to an additional systematic effect is added to YF_3 to get a higher level of correction (YF_4 methodology). This additional correction accounts for variations in the reflection coefficient of the noise source between the hot and cold states ($\Gamma s\_c \neq \Gamma s\_h$). Results from YF_4 approach are shown in Fig. 5. Systematic error at high gain is virtually eliminated and a noticeable increase in the uncertainty can be signaled.

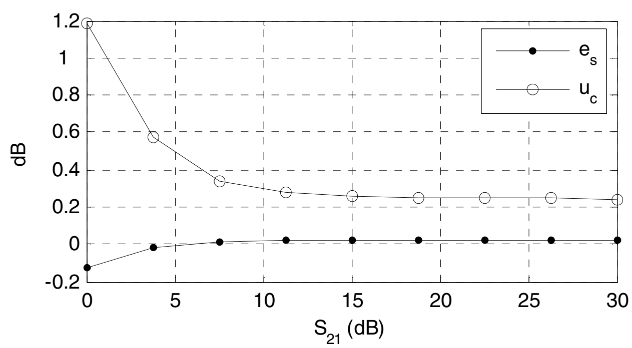

Fig. 5. *Y-factor* based methodology YF_4. Corrections account for second stage noise, DUT available gain, receiver noise figure variation with source termination and changes in the reflection coefficient of the noise source ($\Gamma s\_c \neq \Gamma s\_h$).

This last result suggests that, in our particular case, systematic error at high gain is mainly due to the changes of the noise source reflection coefficient from hot to cold states. In fact, let us consider a methodology YF_5 consisting in adding a single correction term accounting for these changes to the classical YF_1. Results depicted in Fig. 6 confirm that YF_5 gets the same negligible systematic error than YF_4 at high gain. However, the extra correction terms of YF_4 methodology require non-trivial calibrations (noise calibration of the receiver for example) that complicate and slow down the whole measurement process. Part of these calibrations and extra measurements are avoided in YF_5 which is a simpler methodology than YF_4. Therefore, for these particular setup characteristics, YF_5 can be a judicious strategy if we are only concerned with the noise figure measurement of high gain amplifiers.

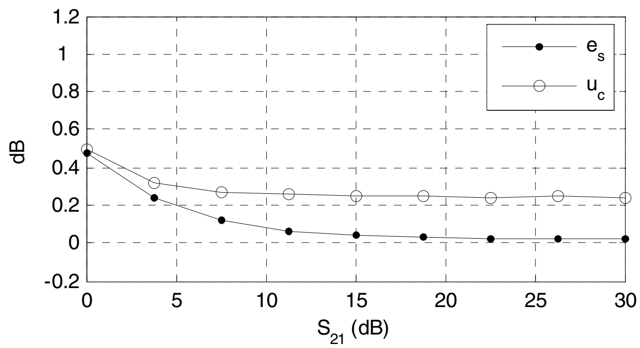

Fig. 6. *Y-factor* based methodology *YF_5*. Corrections only account for second stage noise and changes in the reflection coefficient of the noise source ($\Gamma s\_c \neq \Gamma s\_h$).

The changes in the noise source reflection coefficient being critical in our example, let us consider now a *cold-source* based methodology (*CS*). By nature, *cold-source* methodologies minimize the effects of reflection coefficient variations since, during DUT measurement, the noise source only operates at its cold state. The methodology analyzed here adds to the basic formulation (4) all the available correction terms. Results plotted in Fig. 7 show a negligible systematic error for all the gain range. This result can be compared with the one obtained in Fig. 5 for *Y-factor* based methodology *YF_4*, which has a comparable level of correction terms. It can be observed that *CS* improves the systematic error at low gain with respect to *YF_4*. In addition, *CS* presents a significantly lower uncertainty for low DUT gain, although a little higher for high gain amplifiers.

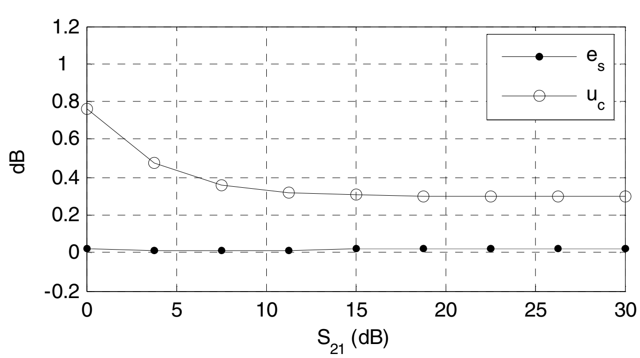

Fig. 7. *cold-source* based methodology *CS*. Corrections account for second stage noise, DUT available gain and receiver noise figure variation with source termination.

It is important to remark that most of the results discussed in these examples rely on the independent handling of systematic error and uncertainty. Actually combining both in a single quantity, as in [3], can result in a loss of valuable information at the time of making decisions about the methodology to use, the design of the setup, the hardware implementation, the importance of a particular systematic effect for achieving a required accuracy, etc.

VI. CONCLUSION

An uncertainty estimation tool applied to noise figure calculation of microwave circuits has been developed in this paper. The tool is specially designed to allow comparative analysis of different methodologies with different level of corrections for systematic effects. It can be used to extract reliable information about the suitable strategy and formulation to be used, the hardware characteristics of the setup, the real benefits of each systematic error correction or even the required balance between measurement accuracy and measurement complexity. Examples carried out on a 44 GHz low noise amplifier for radio astronomy applications, in a Q-band measurement setup, are given to illustrate the potentialities of the tool.

ACKNOWLEDGMENT

The authors wish to thank the University of the Basque Country for partially funding this work.

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